Registration of multiple Low resolution images for motion using hybrid approach.

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***Abstract:** A variety of image registration methods have been derived for applications such as image analysis, fusion, compression.. Robust registration is a required for successful multiple image processing for super-resolution. Image registration algorithms are classified as either global or feature-based, or based on being performed in either the Fourier or spatial domain. Spatial domain techniques are used for situation requiring accurate estimation of sub-pixel motion. A Gaussian pyramid approach is standard method to extend the region of convergence of spatial domain techniques, but often rely on the a priori information. Therefore are limited in dynamic range of the global relative motions. This paper explores frequency and spatial domain registration techniques and analyze for potential benefits. Fourier domain-based correlation techniques are fast and accurate for estimates of motion parameters like shifts, rotation, zoom etc, and perform well for a large range of motion magnitudes. It is concluded that, performance of Kerens algorithms is found better than vandewalle as it is an iterative algorithm find the rotation angle by using a 3-parameter affine transformation generated using Gaussian pyramid.*

(Keywords: Super resolution, Registration, Blurring)

I- Introduction: The aim of super resolution is to increase the resolution which is a measure of frequency content in an image. High-resolution (HR) images consist of larger frequency range as compare to low-resolution (LR) images. In applications where need to extract more information from images, the more detailed images are the better. However, details are limited by hardware as it is expensive and hard to obtain [1]. Super-resolution (SR) is bandwidth extrapolation beyond the pass band of the imaging system as additional spatio-temporal information available in the sequence of low-resolution images used for reconstruction at resolutions higher than that of the original data [3].

The direct approach to increase spatial resolution is to increase the number of pixels per unit area but it leads to shot noise. Another way is to increase the chip size, which results in an increase in capacitance [2]. Therefore, a new approach to increase spatial resolution is by using use signal processing techniques to obtain an HR image from observed multiple low-resolution (LR) images or single LR image.

In super resolution of an image, it is assumed that several LR having sub pixel variation between them can be combined into a single HR image: by decreasing the time resolution, and increasing the spatial frequency content [2]. There must be some LR images, consist of translational, rotation, zoom or different viewing angles which ensure the information contained about the object in multiple images. We need the knowledge of transformations between the LR images, which enable us to obtain a much better resolved image of the object. As super-resolution reconstruction is an ill-posed inverse problem, due to a multiple possible solutions exists for given a set of LR images so, popular approach is to constrain the solution space according to a-priori information about the solution[3]. If scene motions are estimated within subpixel accuracy, then by combining these LR images, SR image reconstruction is possible by the way as shown in Figure 1.

In the process of capturing digital image, it suffers from blur, noise, and aliasing effects and the aim of SR reconstruction is to restore HR image from many degraded and aliased LR images. The combine information from various LR observations of the same scene allows us SR reconstruction of the scene. Observation model relate LR images to SR image which can be implemented with various ways of processing with different success ratio for different types of images.

$$y_k = D B_k M_k + n_k \quad \text{for } 1 \leq k \leq p \quad \text{----(1)}$$

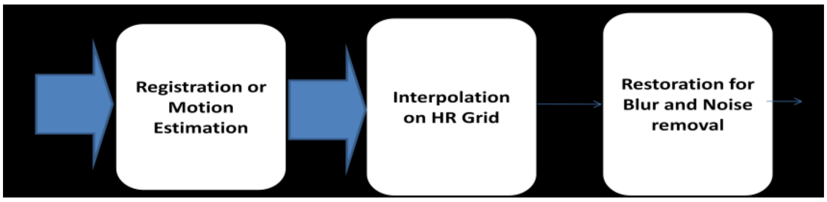

Figure1: Scheme for super resolution [2]

Where M_k is a warp matrix of size L_1N_1 X L_2N_2, B_k represents a L_1N_1 X L_2N_2 blur matrix, D is a $(N_1 \ N_2)^2$ x L_1N_1 X L_2N_2 subsampling matrix, and n_k represents a lexicographically ordered noise vector. The motion that occurs during the image acquisition is represented by warp matrix M_k . By discrediting a continuous warped, blurred scene can express these models without loss of generality as follows:

$$y_k = W_k x + n_k \ , k = 1, \ldots, p, \quad \text{------ (2)}$$

Where matrix W_k of size $(N_1 \ N_2)^2$ x L_1N_1 X L_2N_2 represents, via blurring, motion, and subsampling, the contribution of HR pixels in x to the LR pixels in y_k (2).LR images are sub sampled as well as shifted with subpixel precision. If the LR images have different subpixel shifts from each then, the new information present in each LR image can be utilized to obtain

HR image. Quality of HR image obtained can be improved if the relative displacement in LR images is estimated accurately, and some knowledge of the image processing is available. Success of SR depends on image registration (motion estimation) accuracy in terms of the accurate measure of sub-pixel shift. Any tiny error in estimation of such shift leads to an incorrect HR image.

From the observations the registration techniques are classified as; frequency domain or spatial domain. Frequency domain methods utilize the shifting property of the Fourier transform to obtain global translational scene motion, and use the sampling theory to ensure the effective restoration by the availability of multiple observation images. For global translational motion, frequency domain methods are computationally attractive. Tsai and Hunag were the first to consider the problem in 1984 [4]. With an approach, by formulating a set of equations in the frequency domain, by using the shift property of the Fourier transform. A finite object imaged by diffraction limited system can be perfectly resolved by extrapolation in frequency domain [5]. Spatial Domain techniques utilizes General observation models, which may include:– Arbitrary motion models (global or non-global)– Motion blurring due to non-zero aperture time, optical system degradations, non-ideal sampling. Spatial methods have ability to model complex degradations these methods have inclusion of a-priori constraints are – Spatial domain image models such as Markov Random Fields, Set based constraints (POCS formulation) and Nonlinear models.

II- Registration techniques: This section consists deals with four registration methods. . The methods discussed are proposed by Vandewalle , Marcel, Keren and Lucchese respectively. The methods discussed are frequency and spatial domain, but spatial domain needs priory information else the results are poor and large computations are needed with greater accuracy for estimation.

A) Vandewalle registration :_Vandewallle et al [19]approach consist of planar motion estimation for registration of aliased LR images using Fourier transformation to estimates the motion with sub-pixel precision to estimate the motion parameters between the reference LR and the other LR images. The motion is expressed by three parameters: horizontal and vertical shifts ($\Delta X_{h,}$ ΔX_{v}) and rotation angle φ. A reference LR image f1(x) and its shifted and rotated version f2(x) has magnitudes of their Fourier transform (FT) as |F1(u)| and |F2(u)| . |F2(u)| is a rotated version of |F1(u)| over the same angle φ as the spatial domain rotation but are not affected by horizontal and vertical shifts , because the spatial domain shifts only affect the phase values of the Fourier transforms. Need to estimate first the rotation angle φ from the magnitudes of the Fourier and compensation for the rotation. Use these compensated images to determine the shifts from the phase difference between FT of compensated LR images [19]. The shift parameters is estimated using the slope of the phase difference ∠ (F2(u)/F1(u)). For the solution to be less sensitive to noise, a plane is fitted through the phase difference using a least squares method.

A HR image is reconstructed from a set of M LR images f_{LR},m (m = 1, 2, ... ,M) with Fourier transform F_{LR},m. Compute the Fourier transforms F_{LR},w,m of all LR images. The rotation angle between every image f_{LR},w,m (m = 2, ... ,M) and the reference image f_{LR},w,1 is estimated. Then rotate image f_{LR},w,m by −φm to cancel the rotation for accurate Shift estimation, then horizontal and vertical shifts between every image f_{LR},w,m (m = 2, ... ,M) and the reference image f_{LR},w,1 are estimated. To estimate the phase difference between image m and the reference image as ∠(F_{LR},w,m/F_{LR},w,1). Let assume we have a continuous two-dimensional reference signal $f_0(x)$ and its shifted and rotated version $f_1(x)$ i.e. $f_1(x) = f_0(R(x + x1))$,

with $x = \begin{pmatrix} x_h \\ x_v \end{pmatrix}, x_{1h} = \begin{pmatrix} x_h \\ x_v \end{pmatrix} \;, \begin{pmatrix} \cos\theta_1 & -sin\theta_1 \\ sin\theta_1 & cos\theta_1 \end{pmatrix}$ ------ (4)

|F0(u)| and |F1(u)| do not depend on the shift values x1, because the spatial domain shifts only affect the phase values of the Fourier transforms. Therefore we can first estimate the rotation angle θ1 from the amplitudes after compensation for the rotation, the shift can be computed from the phase difference. The rotation is computed as the angle θ1 for which the Fourier transform of the reference image |F0(u)| and the rotated Fourier transform of the image to be registered |F1(Ru)| have maximum correlation. FT magnitudes are transformed in polar coordinates; the rotation is reduced to a (circular) shift by same angle θ1. This requires a transformation of the spectrum to polar coordinates. Initially, compute the frequency content H as a function of the angle θ by integrating over radial lines:

$H(\theta) = \int_{\theta-\Delta\theta/2}^{\theta+\Delta\theta/2} \int_0^{\infty} |F(u_r \quad u_\theta)| \, du_r du_\theta$ ------ (5)

The discrete function H(θ) is the average of the values on the rectangular grid that have an angle θ−Δθ/2 < uθ < θ +Δθ/2. As we need to compute the rotation angle with a precision of 0.1 degrees, H(θ) is determined after every 0.1 degrees, to have same number of sample values at every angle, the average is only evaluated on a circular disc of values for which ur < ρ (where ρ is the image radius, or half the image size). At the end, as the values for low frequencies are very large compared to the other values and are very coarsely sampled as a function of the angle, we discard the values for which ur < ϱρ, with ϱ = 0.1. Method works best if strong directionality is present in the images. Now shift parameters are computed as the slope of the phase difference ∠(F1(u)/F0(u)) and to have the solution less affected by noise, fit a plane through the phase differences using a least squares method. Due to the aliasing, the relations between the Fourier transforms of the images are no longer valid. For the low frequencies k, with |k| < N − K, only a single term in the sums is non-zero, and Y1(k)/Y0(k) is still linear phase, shift can therefore be estimated from the low, aliasing-free frequencies. However, the high frequency part of a signal typically has a lower signal-to-noise ratio, and in under sampled signals suffer from aliasing so we discard the high frequency components in this registration algorithm to avoid effect of aliasing on estimation.

B) Marcel registration [16]: Marcel method based on the phase correlation function. Let the image I_2 be a shifted version of the image I_1 by (x0,y0),

$I_2(x \quad y) = I_1(x - X0\,, \quad y - y0)$---- (6)

After computing the FT of both LR images, the following relationship due to the shift property of the FT is utilized $I_2(u \quad v) = I_1(u\,, \quad v)\, e^{-j(ux0+\ vx0)}$---- (7)

As, a shift in the spatial domain produces a phase difference in the frequency domain, the normalized cross power spectrums finally denudes as,

$\frac{I_2(u \quad v)\, I_1(u\,, \quad v)}{|I_2(u \quad v)\, I_1(u\,, \quad v)|} = e^{-j(ux0+\ vx0)}$ --------------(8)

The (PC) function is finally obtained by taking the Inverse Fourier Transform (IFT) of the cross-power spectrum, which gives a δ (xo,yo) as result but the accuracy of the approach is very poor and not acceptable for SR reconstruction

C) Keren's registration: It is hybrid method consist of combination of Fourier and spatial domain approch [10]. Prior to the algorithm need to discuss the pyramid generation for iterative solution of motion estimation. There are two main ways to obtain the pyramids i.e. lowpass and band pass. A lowpass pyramid is obtain by smoothing the image with smoothing filter and then subsampling the smoothed image, mostly by a factor of two along each coordinate. The resulting image is then subjected to the same procedure, and the cycle is repeated multiple times, where each cycle of process results in a smaller image with increased smoothing, and decreased spatial sampling density, the entire multi-scale representation is like a pyramid, with the original image at the bottom and each cycle's smaller image placed on top, different smoothing kernels are proposed for generating pyramids. Keren, Peleg and Brada, proposed an iterative registration process to measure translation and rotation to sub-pixel accuracy using a Gaussian pyramid technique obtain a series of images which are weighted down using a Gaussian average. The error between two images f and g after rotation and translation is expressed by Taylor expressions:

$$E(a,b,\theta) = \sum \left[f(x,y) + \left(a - y\theta - \frac{x\theta^2}{2}\right)\frac{\delta f}{\delta x} + \left(b + x\theta - \frac{y\theta^2}{2}\right)\frac{\delta f}{\delta y} - g(x,y) \right]^2 \text{------------(9)}$$

Where a is a horizontal translation, b a vertical translation and rotation θ. The error is minimize iteratively by first moving the second image g by the accumulated translation and rotation value, then re-estimating the difference between the two images. The implementation evaluated with Gaussian pyramid images from top to bottom of stack to increase the speed and robustness and accuracy at the end.

An iterative planar motion estimation algorithm based on Taylor expansions, is getting movement parameter with high sub pixel precision in cases of small angle. It is a hybrid method where Gaussian pyramid approach is used to extend the region of convergence of spatial domain techniques. On the contrary, Fourier based correlation techniques, the log-polar FFT (L-P FFT) method provide fast and reasonably accurate estimates of global shifts, rotation, and scale change, and perform well over a large range of motion magnitudes. This technique is able to estimate global x- and y-axis translations, rotation, and uniform scale change as described by affine coordinate transformation between two images f1 and f2 as:

$$f_2(x,y) = f_1\left(xs\cos\theta_0 + ys\sin\theta_0 + t_x, -xs\sin\theta_0 + ys\cos\theta_0 + t_y\right) \text{------------(10)}$$

Where t_x and t_y represent translations along the x- and y-axis, respectively, θ_0 is rotation, and s is the uniform scale (zoom) factor applied to both axes. The algorithm utilizes three separate correlations to estimate θ_0,t_x and t_y and need to compute FFT of each images- F1, F2 and need only the magnitudes | F1| and| F2| as translation invariant. Then map the magnitudes from Cartesian to log-polar coordinates using bilinear interpolation as M1,M2 respectively and find the correlation between M1(log ρ, θ) and M2(log ρ − log s, θ − θ_0). Determine the peak in the correlation, the location of peak estimates of rotation θ_0 and scale s. Create two rotated and scaled versions of the "zoomed" image, using (s,θ_0) and (s,θ_0+180) and compute global correlations between the baseline image and the two scaled and rotated images given the two possible solutions for rotation angle.

The algorithm is further modified by using Fourier interpolation which improve the correlation peak location estimation, which results in improved motion estimates. For modification Taylor series 3-parameter affine models are employed, the technique was proposed for multiple image resolution as it provides robust and accurate estimation of subpixel motion parameters. But, the technique is limited in dynamic range of the global motion estimates. The iterative spatial domain approach is based on a Taylor series expansion of the translated and rotated image which assumes a 3-parameter affine transformation, resulting in the approximation for sum squared error E (t_x, t_y , θ_0) between the two LR images is approximated using the equation (11) and solving for the minimum of error and discard the higher order terms to obtain the system of linear equations for avoid complexity.

$$\begin{bmatrix} \sum\left(\frac{\delta f_1}{\delta x}\right)^2 & \sum\frac{\delta f_1}{\delta x}\frac{\delta f_1}{\delta y} & \sum\frac{\delta f_1}{\delta x}\left(x\frac{\delta f_1}{\delta y}-y\frac{\delta f_1}{\delta x}\right) \\ \sum\frac{\delta f_1}{\delta x}\frac{\delta f_1}{\delta y} & \sum\left(\frac{\delta f_1}{\delta y}\right)^2 & \sum\frac{\delta f_1}{\delta y}\left(x\frac{\delta f_1}{\delta y}-y\frac{\delta f_1}{\delta x}\right) \\ \sum\frac{\delta f_1}{\delta x}\left(x\frac{\delta f_1}{\delta y}-y\frac{\delta f_1}{\delta x}\right) & \sum\frac{\delta f_1}{\delta y}\left(x\frac{\delta f_1}{\delta y}-y\frac{\delta f_1}{\delta x}\right) & \sum\left(x\frac{\delta f_1}{\delta y}-y\frac{\delta f_1}{\delta x}\right)^2 \end{bmatrix} \begin{bmatrix} t_x \\ t_y \\ \theta_0 \end{bmatrix} = \begin{bmatrix} \sum\left(\frac{\delta f_1}{\delta x}\right)(f_1-f_2) \\ \sum\left(\frac{\delta f_1}{\delta y}\right)(f_1-f_2) \\ \sum\left(x\frac{\delta f_1}{\delta y}-y\frac{\delta f_1}{\delta x}\right)(f_1-f_2) \end{bmatrix} \text{-----(11)}$$

Solution of above equation is valid for small values of tx, ty, and θ_0 and an iterative approach is recommended to solve it, the shifted and rotated image is corrected based on initial solution. The process is repeated until convergence occurs.

D) Lucchese registration –Estimation of shift and rotation parameters using Lucchese and Cortelazzo algorithm [15], is based on phase correlation method and estimates planar roto-translations in the frequency domain . This methods utilize all the image information, results in robust against noise, and t can be very accurate. Frequency domain approach is used for rigid motion, for robustness to noise, and separability of the rotational and translational motions. Initially estimate the rotational motion from the magnitudes of the FT and then compensate for rotation. Use compensated output for further estimation of the translational motion. For two rotated and translated LR images, the difference between the FT magnitude of one image and the mirrored version, w.r.t. both frequency axis will have a pair of orthogonal zero-crossing lines and these lines are rotated about the axes by an angle half the rotational angle between two images. So , the estimation of a rotation is nothing but to determination of two zero-crossing lines, the estimation of the rotation by a three-stage coarse-to-fine procedure that results in a precision of up to hundredths of a degree.

The two zero-crossing lines are very robust to noise at low frequencies and the given approach avoids polar coordinates. As estimation detect two zero-crossing lines, from the difference of two normalized DFT magnitudes need to use of zero-padding to increase the resolution of the two DFT magnitudes and ultimately resolution of the two lines, at the cost of computational cost [11]. To limit this drawback first downsize the images to half and then zero-pad it to obtain two new images having twice the size of the original ones which reduces the computation. The estimates of θ is a three stage process as [15]:

Stage 1—Coarse Estimate θ_c of θ:

As required lines pass through the origin, need to find only accumulation point of the angular coordinate that defines the slope of the lines. To obtain it Cartesian coordinates Kx and Ky of locus's points transformed into the polar coordinates and build a histogram for the distribution of the angles, such a histogram H(φ) is will show a prominent peak corresponding to the slope of lines.

By Hermitian symmetry of the Fourier transform, build the histogram H(φ) by ,histogram $H_1(\varphi)$ of the angles $-\pi/4 <= \varphi < \pi/4$and a histogram $H_2(\varphi^T)$ of the angles $\pi/4 <= \varphi^T < 3\pi/4$and sum them up as $H(\varphi) \approx H_1(\varphi) + H_2(\varphi^T)$. In absence of noise, the two orthogonal lines appear over a wide range of frequencies; with noise lines start curling and shifting at high frequencies, which limit the histogram H(φ) construction within the region ρ <= Rmax a preselected threshold and the first estimate of rotation angle θ as

$$\dot{\theta}_c = \arg\max_{\theta}\{H(\theta)\}$$

Stage 2—Refinement $\theta_c^!$ of θ_c :

As the rough guess of θ is available, use it to weight the contribution of the angles φ to $H_1(\varphi)$ and the contribution of the angles φ^T to $H_2(\varphi^T)$ according to their closeness to the first coarse estimate.

1. From stage 1 keep points satisfying constraint $\varphi_c - \gamma \le \varphi \le \varphi_c + \gamma$
2. Build histograms $H'_1(\varphi)$ and $H'_2(\varphi^T)$ and add to obtain $H'(\theta)$.
3. Compute $H'(\theta) = H'(-2\varphi)$.
4. Normalize the result to obtain $h'(\theta) \triangleq \frac{H'(\theta)}{\sum H'(\theta)}$ and find refined $\dot{\theta}'_c$of $\dot{\theta}_c$ as a mean of $h'(\theta)$ and it's variance.

the stage 2 estimate θ'_c is the mean of the pdf $h'(\theta)$ in place of position of the peak of $H'(\theta)$ because there might exist several peaks clustered together.

Stage 3: fine estimation of θ_f of θ :- fine estimation of θ by using linear regression technique, with help of least squares algorithm is based on fitting two orthogonal lines to the locus points laying close to angular direction $-\dot{\theta}'_c$ and $\frac{\pi}{2} - \dot{\theta}'_c$. Let $\varphi'_c = -\dot{\theta}'_c$ and $\varphi'^T_c = \frac{\pi}{2} - \dot{\theta}'_c$, and let Ω be set of locus points laying within given two angular sectors φ'_c and $\varphi'_c + \pi$ with margin of +/- σ_θ , similarly Ω^T for φ'^T_c. then minimize mean square error $\varepsilon^2(\varphi)$ MSE with respect to φ . which is obtained by fitting lines $k_x \sin\varphi - k_y \cos\varphi = 0$ to points of Ω and $k_x \cos\varphi + k_y \sin\varphi = 0$ to points of Ω^T as $\dot{\varphi_f} = \arg\min_\varphi \varepsilon^2(\varphi)$ solution by equating $\frac{d\varepsilon^2(\varphi)}{d\varphi}$ as

$$\theta_f = \frac{1}{2}\arctan\left(\frac{2\left(\frac{1}{N}\sum kx\,ky \;\; \frac{1}{N^T}\sum\sum kx\,ky\right)}{\left(\frac{1}{N}\sum(kx^2 - ky^2) - \frac{1}{N^T}\sum(kx^2 - ky^2)\right)}\right) \quad \text{------------(14)}$$

III- Results: : All four algorithms are implemented and tested for various LR images which are rotated and or shifted with known value and results of various algorithms are compared for accurate determination of subpixel shift between two images. Result table gives registration performance of the algorithms for rotation only or rotation and shift together. Result table shows sampled results for LR images for Image 4, image 5 and image 7 i.e. (LR1, LR2, LR3 , LR4)

Table 1 : Rotation and shift along spatial coordinates given to LR images

	Image4				Image5			Image7		
Algorithm	image	rotation	x	Y	rotation	x	y	rotation	x	y
Given	LR2	-0.0015	-0.0015	-0.0025	-0.002	-0.001	-0.002	0.0015	0.0012	0.0016
Vandewalle	LR2	0	-0.000537	-0.001849	0	-0.00047	-0.00383	0	0.001257	0.002075
Keren	LR2	-0.00075	-0.001013	-0.002073	-0.00119	-0.00049	-0.00296	0.001241	0.001116	0.001541
Marcel	LR2	0	0	0	0	0	0	0	0	0
Lucchese	LR2	0.000573	0	0	-0.0061	0	0	0.004829	0	0
Given	LR3	-0.0025	-0.0025	0.003	-0.006	0.002	0.001	-0.0015	-0.0018	0.0022
Vandewalle	LR3	0	-0.003513	0.0050158	-0.1	-0.0363	0.014249	0	-0.0018	0.001557
Keren	LR3	-0.00224	-0.002218	0.0031168	-0.0037	0.003033	0.000694	-0.00151	-0.00214	0.002047
Marcel	LR3	0	0	0	0	0	0	0	0	0
Lucchese	LR3	-0.00605	0	0	-0.011	0	0	0.002311	0	0
Given	LR4	0.0025	0.0015	0.0015	-0.006	-0.002	0.002	-0.002	-0.0024	-0.0008
Vandewalle	LR4	-0.1	-0.020447	-0.0007841	-0.1	-0.03778	0.015401	0	-0.00303	-0.00109
Keren	LR4	0.001399	0.0018731	0.0017084	-0.00424	-0.00132	0.001518	-0.00185	-0.00282	-0.0008
Marcel	LR4	0	0	0	0	0	0	0	0	0
Lucchese	LR4	-0.00723	0	0	-0.01477	0	0	0.003749	0	0
Keren	LR4	0.002191	-1.63E-04	-2.37E-04	-0.00213	0.000821	-0.00031	0.000256	2.08E-05	1.98E-05
Marcel	LR4	0	0	0	0	0	0	0	0	0
Lucchese	LR4	0.002094	0	0	-0.0128	0	0	0.005391	0	0

			
Image4 LR1	**LR2**	**LR3**	**LR4**
			
Image5 LR1	**LR2**	**LR3**	**LR4**

			
Image7 LR1	**LR2**	**LR3**	**LR4**

LR images: Table 3 : LR images Rotated and shifted along spatial coordinates (LR2,3,4 are rotated and shifted w.r.t. LR1)

Table 2 : Only Rotation given to LR images

	Image4				**Image5**			**Image7**		
Algorithm	image	rotation	x	y	rotation	x	y	rotation	x	y
Given	LR2	0.004	0	0	-0.002	0	0	0.0001	0	0
Vandewalle	LR2	-0.1	-0.021022	-0.004331	0	0.00059	-0.00025	0	0.000267	-6.35E-05
Keren	LR2	0.002556	-3.67E-04	-1.31E-04	-0.00098	0.000356	-0.00019	8.06E-05	2.33E-05	9.74E-06
Marcel	LR2	0	0	0	0	0	0	0	0	0
Lucchese	LR2	0.000954	0	0	-0.00647	0	0	0.005348	0	0
Given	LR3	0.005	0	0	-0.003	0	0	0.0002	0	0
Vandewalle	LR3	-0.1	-0.021038	-0.005602	-0.1	-0.0379	0.011701	0	0.00036	-0.0002
Keren	LR3	0.003405	-4.60E-04	-1.64E-04	-0.00177	0.000635	-0.00031	0.000148	2.61E-05	-1.03E-05
Marcel	LR3	0	0	0	0	0	0	0	0	0
Lucchese	LR3	0.002267	0	0	-0.00899	0	0	0.005409	0	0
Given	LR4	0.003	0	0	-0.004	0	0	0.0003	0	0
Vandewalle	LR4	-0.1	-0.020891	-0.00393	-0.1	-0.03742	0.01185	0	0.000312	-0.00025

			
Image4 LR1	**LR2**	**LR3**	**LR4**
			
Image5 LR1	**LR2**	**LR3**	**LR4**
			
Image7 LR1	**LR2**	**LR3**	**LR4**

Table 4 : Only Rotation given to LR images LR images Rotated (LR2,3,4 are rotated w.r.t. LR1)

IV) Conclusion: The result table concludes that, performance of Marcel way of registration is worst and can not used for super resolution as subpixel shift or small rotation can not be identified by this algorithm. The Lucchese approach determines the rotation accurately but it does not calculate shift so need to go for another algorithm for shift measurement ultimately accuracy of result depends on combination. Vandewalle approach and keren hybrid method these two algorithms provide accurate shift and rotation of LR images in terms of subpixel shift. Performance of Kerens algorithms is found better than vandewalle as it is an iterative algorithm find the rotation angle by using a 3-parameter affine transformation generated using Gaussian pyramid, resulting in the following approximation for sum squared error E (t_x, t_y , θ_0) between the two images can be approximated using above equation and solving for the minimum of E and ignoring higher order terms resulting in system of linear equations. The accurate determination of angle reflects in accurate measurement of shift which by same way in both algorithms.

The average percentage error in determination of registration parameter in keren's method is very small (25% to 35%) for rotation detection which result into registration of pixels at the right position between two pixels of LR images. The error is further reduces to (1% to 5%) in determination of x and y subpixel movement which is better than other methods. These small errors will place the subpixel at the correct position on a regular high resolution grid for super resolution and the error effect is not observed as with error the pixel still lays in between the two pixels on regularly sampled HR grid.

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